Name: _____ Date:_____

PC: Rational Zeros Theorem

Rational Zeros Theorem

If the polynomial $P(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_0$ has integer coefficients, then every rational zero of P is of the form

$$\frac{p}{q}$$

where p is a factor of the constant coefficient a_0

and q is a factor of the leading coefficient a_n

1. List all possible rational zeros of $P(x) = x^3 - 3x + 2$.

2. Find the zeros of $P(x) = x^3 - 3x + 2$

3. Factor the polynomial $P(x) = 2x^3 + x^2 - 13x + 6$

4. Let $P(x) = x^4 - 5x^3 - 5x^2 + 23x + 10$
   (a) Find the zeros of $P(x)$.
   (b) Sketch the graph of P **without using your calculator.**

Exercises

1–6 ■ List all possible rational zeros given by the Rational Zeros Theorem (but don't check to see which actually are zeros).

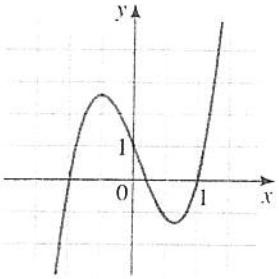
1. $P(x) = x^3 - 4x^2 + 3$
2. $Q(x) = x^4 - 3x^3 - 6x + 8$
3. $R(x) = 2x^5 + 3x^3 + 4x^2 - 8$
4. $S(x) = 6x^4 - x^2 + 2x + 12$
5. $T(x) = 4x^4 - 2x^2 - 7$
6. $U(x) = 12x^5 + 6x^3 - 2x - 8$

7–10 ■ A polynomial function P and its graph are given.

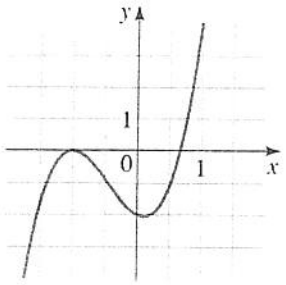
(a) List all possible rational zeros of P given by the Rational Zeros Theorem.

(b) From the graph, determine which of the possible rational zeros actually turn out to be zeros.

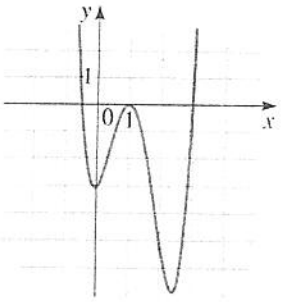
7. $P(x) = 5x^3 - x^2 - 5x + 1$



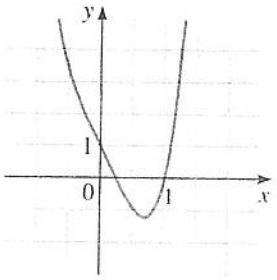
8. $P(x) = 3x^3 + 4x^2 - x - 2$



9. $P(x) = 2x^4 - 9x^3 + 9x^2 + x - 3$



10. $P(x) = 4x^4 - x^3 - 4x + 1$



11–40 ■ Find all rational zeros of the polynomial.

11. $P(x) = x^3 + 3x^2 - 4$
12. $P(x) = x^3 - 7x^2 + 14x - 8$
13. $P(x) = x^3 - 3x - 2$
14. $P(x) = x^3 + 4x^2 - 3x - 18$
15. $P(x) = x^3 - 6x^2 + 12x - 8$
16. $P(x) = x^3 - x^2 - 8x + 12$
17. $P(x) = x^3 - 4x^2 + x + 6$
18. $P(x) = x^3 - 4x^2 - 7x + 10$
19. $P(x) = x^3 + 3x^2 + 6x + 4$

20. $P(x) = x^3 - 2x^2 - 2x - 3$

21. $P(x) = x^4 - 5x^2 + 4$

22. $P(x) = x^4 - 2x^3 - 3x^2 + 8x - 4$

23. $P(x) = x^4 + 6x^3 + 7x^2 - 6x - 8$

24. $P(x) = x^4 - x^3 - 23x^2 - 3x + 90$

25. $P(x) = 4x^4 - 25x^2 + 36$

26. $P(x) = x^4 - x^3 - 5x^2 + 3x + 6$

27. $P(x) = x^4 + 8x^3 + 24x^2 + 32x + 16$

28. $P(x) = 2x^3 + 7x^2 + 4x - 4$

29. $P(x) = 4x^3 + 4x^2 - x - 1$

30. $P(x) = 2x^3 - 3x^2 - 2x + 3$

31. $P(x) = 4x^3 - 7x + 3$

32. $P(x) = 8x^3 + 10x^2 - x - 3$

33. $P(x) = 4x^3 + 8x^2 - 11x - 15$

34. $P(x) = 6x^3 + 11x^2 - 3x - 2$

35. $P(x) = 2x^4 - 7x^3 + 3x^2 + 8x - 4$

36. $P(x) = 6x^4 - 7x^3 - 12x^2 + 3x + 2$

37. $P(x) = x^5 + 3x^4 - 9x^3 - 31x^2 + 36$

38. $P(x) = x^5 - 4x^4 - 3x^3 + 22x^2 - 4x - 24$

39. $P(x) = 3x^5 - 14x^4 - 14x^3 + 36x^2 + 43x + 10$

40. $P(x) = 2x^6 - 3x^5 - 13x^4 + 29x^3 - 27x^2 + 32x - 12$

41–50 ▪ Find all the real zeros of the polynomial. Use the quadratic formula if necessary, as in Example 3(a).

41. $P(x) = x^3 + 4x^2 + 3x - 2$

42. $P(x) = x^3 - 5x^2 + 2x + 12$

43. $P(x) = x^4 - 6x^3 + 4x^2 + 15x + 4$

44. $P(x) = x^4 + 2x^3 - 2x^2 - 3x + 2$

45. $P(x) = x^4 - 7x^3 + 14x^2 - 3x - 9$

46. $P(x) = x^5 - 4x^4 - x^3 + 10x^2 + 2x - 4$

47. $P(x) = 4x^3 - 6x^2 + 1$

48. $P(x) = 3x^3 - 5x^2 - 8x - 2$

49. $P(x) = 2x^4 + 15x^3 + 17x^2 + 3x - 1$

50. $P(x) = 4x^5 - 18x^4 - 6x^3 + 91x^2 - 60x + 9$

51–58 ▪ A polynomial P is given.

(a) Find all the real zeros of P.

(b) Sketch the graph of P.

51. $P(x) = x^3 - 3x^2 - 4x + 12$

52. $P(x) = -x^3 - 2x^2 + 5x + 6$

53. $P(x) = 2x^3 - 7x^2 + 4x + 4$

54. $P(x) = 3x^3 + 17x^2 + 21x - 9$

55. $P(x) = x^4 - 5x^3 + 6x^2 + 4x - 8$

56. $P(x) = -x^4 + 10x^2 + 8x - 8$

57. $P(x) = x^5 - x^4 - 5x^3 + x^2 + 8x + 4$

58. $P(x) = x^5 - x^4 - 6x^3 + 14x^2 - 11x + 3$